

# ELEMENTÁRNÍ FUNKCE

## Lineární funkce $y = ax + b$

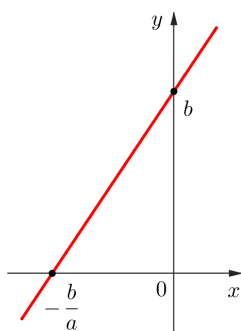
- $a, b \in \mathbb{R}$
- $D(f) = \mathbb{R}$
- graf: přímka

- $a = 0$ : **Konstantní funkce**  $y = b$
- $b = 0$ : **Přímá úměrnost**  $y = ax$

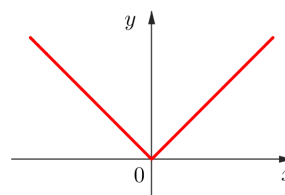
## Absolutní hodnota $y = |x|$

- $D(f) = \mathbb{R}$

Poznámka:  $|a| = \begin{cases} a & \text{pro } a \geq 0 \\ -a & \text{pro } a < 0 \end{cases}$



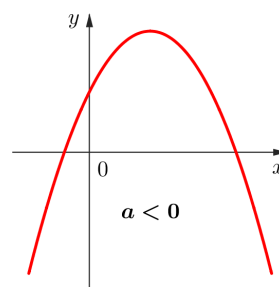
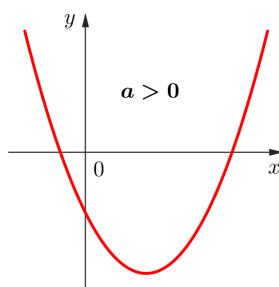
Lineární funkce



Absolutní hodnota

## Kvadratická funkce $y = ax^2 + bx + c$

- $a, b, c \in \mathbb{R}, a \neq 0$
- $D(f) = \mathbb{R}$
- graf: parabola

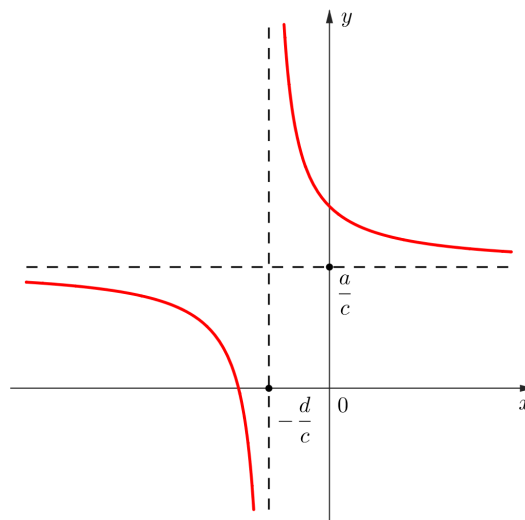


## Lineární lomená funkce $y = \frac{ax + b}{cx + d}$

- $a, b, c, d \in \mathbb{R}, c \neq 0, ad - bc \neq 0$
- $D(f) = \mathbb{R} - \left\{-\frac{d}{c}\right\}$
- graf: rovnoosá hyperbola

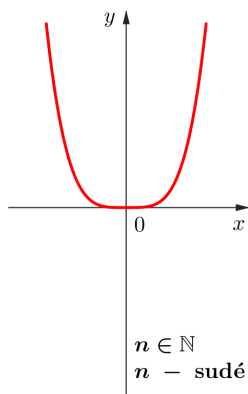
- zvláštní případ:

**Nepřímá úměrnost**  $y = \frac{k}{x}, k \in \mathbb{R} - \{0\}$

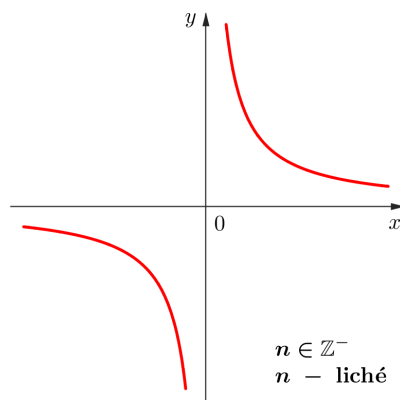
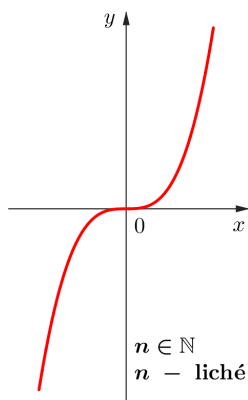
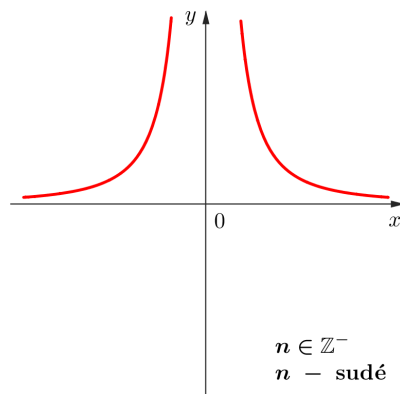


## Mocninná funkce $y = x^n$

- $n \in \mathbb{N}$
- $D(f) = \mathbb{R}$
- graf: parabola  $n$ -tého stupně

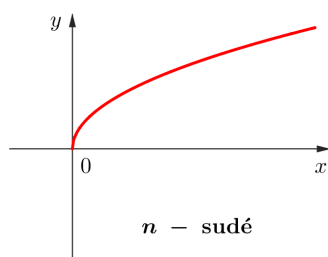


- $n \in \mathbb{Z}^-$
- $D(f) = \mathbb{R} - \{0\}$
- graf: hyperbola  $n$ -tého stupně

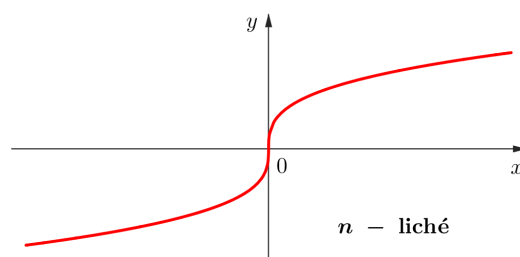


## $n$ -tá odmocnina $y = \sqrt[n]{x}$

- $n \in \mathbb{N}, n \geq 2$
- graf: parabola  $n$ -tého stupně

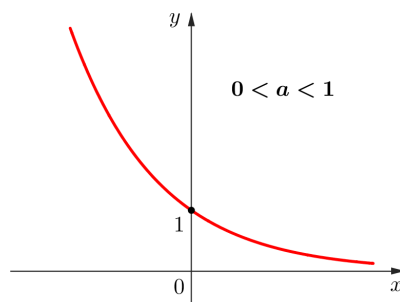
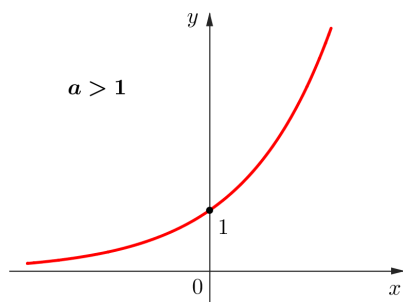


- $n$  sudé ...  $D(f) = \mathbb{R}_0^+$
- $n$  liché ...  $D(f) = \mathbb{R}$



## Exponenciální funkce $y = a^x$

- $a \in \mathbb{R}^+ - \{1\}$
- $D(f) = \mathbb{R}, \quad H(f) = \mathbb{R}^+$



### Důležité vzorce

$\forall x, x_1, x_2 \in \mathbb{R} :$

$$a^{x_1} \cdot a^{x_2} = a^{x_1+x_2}$$

$$\frac{a^{x_1}}{a^{x_2}} = a^{x_1-x_2}$$

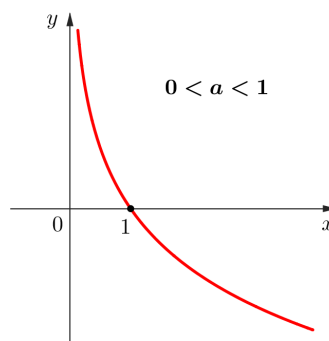
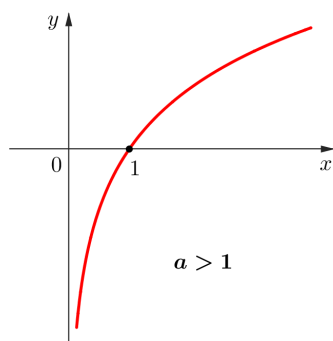
$$(a^{x_1})^{x_2} = a^{x_1 \cdot x_2}$$

$$a^x \cdot b^x = (a \cdot b)^x$$

## Logaritmická funkce $y = \log_a x$

$$\dots \quad a^y = x$$

- $a \in \mathbb{R}^+ - \{1\}$
- $D(f) = \mathbb{R}^+, \quad H(f) = \mathbb{R}$
- **Přirozený logaritmus:**  $y = \ln x = \log_e x, \quad e \doteq 2,71$
- **Dekadický logaritmus:**  $y = \log x = \log_{10} x$



### Důležité vzorce

$\forall x, x_1, x_2 \in \mathbb{R} :$

$$\log_a (x_1 \cdot x_2) = \log_a x_1 + \log_a x_2$$

$$\log_a \frac{x_1}{x_2} = \log_a x_1 - \log_a x_2$$

$$\log_a x^k = k \cdot \log_a x$$

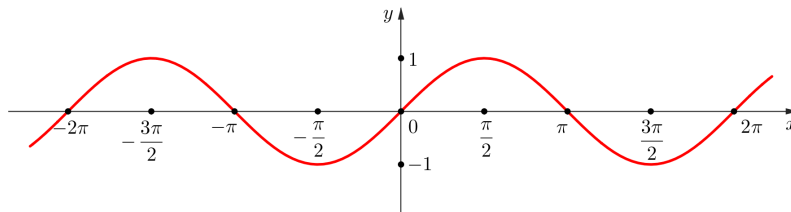
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$\log_a x = -\log_{\frac{1}{a}} x$$

## Goniometrické funkce

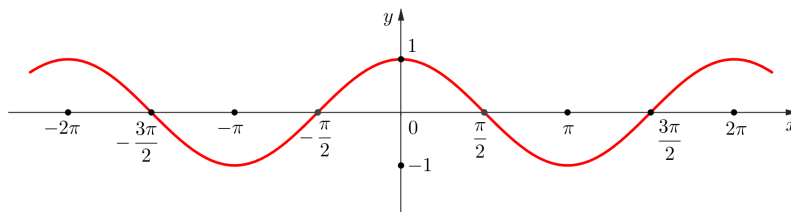
**Sinus**  $y = \sin x$

- $D(f) = \mathbb{R}, \quad H(f) = \langle -1, 1 \rangle$



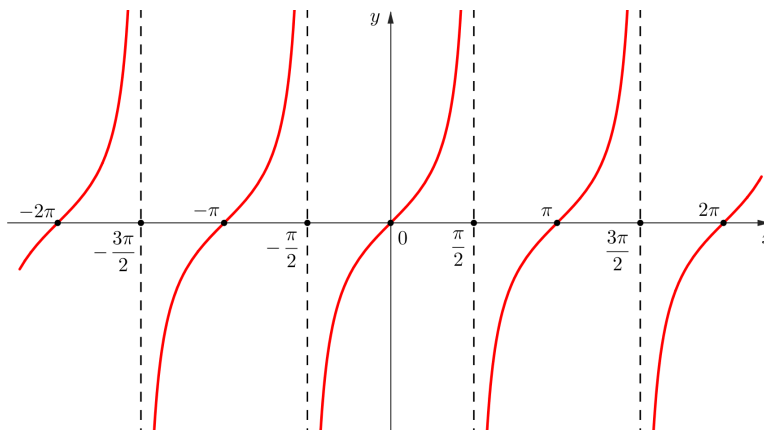
**Kosinus**  $y = \cos x$

- $D(f) = \mathbb{R}, \quad H(f) = \langle -1, 1 \rangle$



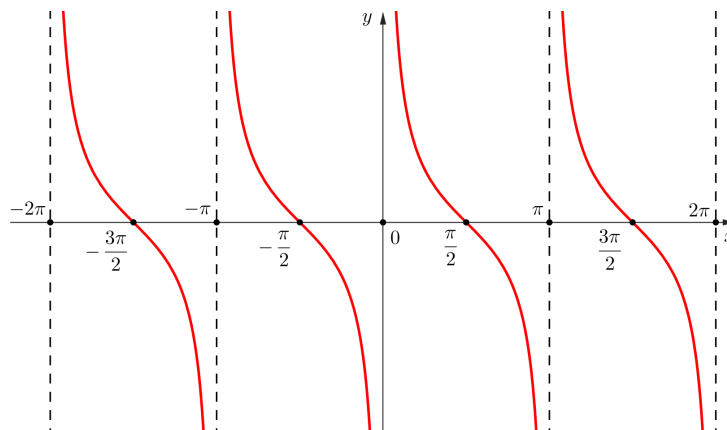
**Tangens**  $y = \operatorname{tg} x = \frac{\sin x}{\cos x}$

- $D(f) = \mathbb{R} - \left\{ (2k+1)\frac{\pi}{2}; k \in \mathbb{Z} \right\}, \quad H(f) = \mathbb{R}$



**Kotangens**  $y = \operatorname{cotg} x = \frac{\cos x}{\sin x}$

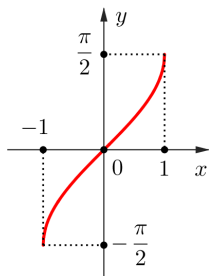
- $D(f) = \mathbb{R} - \{k\pi; k \in \mathbb{Z}\}, \quad H(f) = \mathbb{R}$



## Cyklometrické funkce

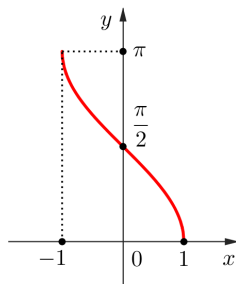
**Arkussinus**  $y = \arcsin x$

- $D(f) = \langle -1, 1 \rangle$ ,  $H(f) = \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle$



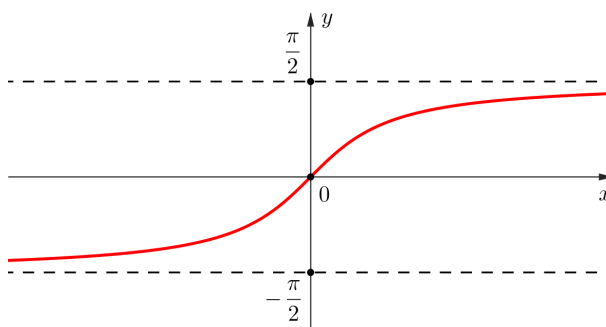
**Arkuskosinus**  $y = \arccos x$

- $D(f) = \langle -1, 1 \rangle$ ,  $H(f) = \langle 0, \pi \rangle$



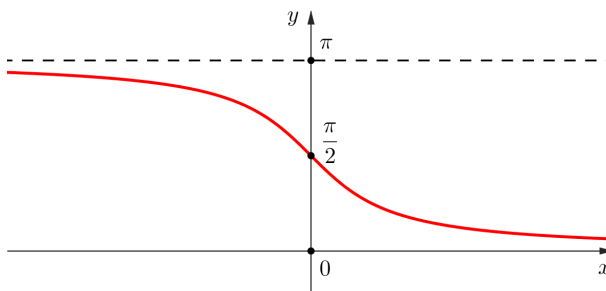
**Arkustangens**  $y = \operatorname{arctg} x$

- $D(f) = \mathbb{R}$ ,  $H(f) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



**Arkuskotangens**  $y = \operatorname{arccotg} x$

- $D(f) = \mathbb{R}$ ,  $H(f) = (0, \pi)$



## Důležité vzorce

$$\sin^2 x + \cos^2 x = 1$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

| $x$                     | 0 | $\frac{\pi}{6}$      | $\frac{\pi}{4}$      | $\frac{\pi}{3}$      | $\frac{\pi}{2}$ | $\pi$ | $\frac{3}{2}\pi$ |
|-------------------------|---|----------------------|----------------------|----------------------|-----------------|-------|------------------|
| $\sin x$                | 0 | $\frac{1}{2}$        | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1               | 0     | -1               |
| $\cos x$                | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$        | 0               | -1    | 0                |
| $\operatorname{tg} x$   | 0 | $\frac{\sqrt{3}}{3}$ | 1                    | $\sqrt{3}$           | /               | 0     | /                |
| $\operatorname{cotg} x$ | / | $\sqrt{3}$           | 1                    | $\frac{\sqrt{3}}{3}$ | 0               | /     | 0                |

| $x$         | 0               | $\frac{1}{2}$   | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1               |
|-------------|-----------------|-----------------|----------------------|----------------------|-----------------|
| $\arcsin x$ | 0               | $\frac{\pi}{6}$ | $\frac{\pi}{4}$      | $\frac{\pi}{3}$      | $\frac{\pi}{2}$ |
| $\arccos x$ | $\frac{\pi}{2}$ | $\frac{\pi}{3}$ | $\frac{\pi}{4}$      | $\frac{\pi}{6}$      | 0               |

| $x$                        | 0               | $\frac{\sqrt{3}}{3}$ | 1               | $\sqrt{3}$      |
|----------------------------|-----------------|----------------------|-----------------|-----------------|
| $\operatorname{arctg} x$   | 0               | $\frac{\pi}{6}$      | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ |
| $\operatorname{arccotg} x$ | $\frac{\pi}{2}$ | $\frac{\pi}{3}$      | $\frac{\pi}{4}$ | $\frac{\pi}{6}$ |